

# Linear and Non-Linear Decoding Techniques in Multi-User MIMO Systems: A Review

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**Abstract:** Multiple input multiple output (MIMO) supports greater data rate and higher reliability in wireless communication. MIMO technique uses the multiple antennas to achieve high transmission rate. The receiver end of the MIMO consists of linear and non-linear decoding techniques. In this paper we are discussing about the linear and non-linear decoding techniques used at the receiver. Zero forcing (ZF), Minimum mean Square error (MMSE), Maximum likelihood (ML), Sphere decoding (SD) are the methods used in wireless communication for reducing the complexity at the receiver.

**Keywords:** Multiple input multiple output, Zero forcing, Minimum mean square error, Maximum likelihood, Sphere decoding.

## 1. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) wireless antenna systems have been recognized as a key technology for future wireless communications. Utilizing the multiple-input multiple-output (MIMO) antenna techniques to achieve high data rate transmission has attracted great attention since the pioneering works [1] and [2]. One common approach to achieve the capacity of MIMO systems is to use spatial multiplexing where streams of independent data are transmitted from the transmitting antennas. These information streams are then separated at the receiver by means of appropriate processing techniques such as maximum likelihood (ML) which achieves optimal performance or linear receivers like Zero-Forcing (ZF). There are various decoding techniques used in a MIMO system and these are discussed here. In linear multiuser detectors, a linear transform is applied to the outputs of conventional matched filters to produce a new set of outputs, which may generate better results. These include the decorrelator [3] and the minimum mean-square error (MMSE) detector [4]. Maximum-likelihood sequence detection (MLSD) is known to have perfect performance on an additive white Gaussian noise (AWGN) channel. However, as the length of a channel increases, the number of states grows exponentially  $L^v$  in Viterbi detector as, where  $L$  is the number of input level and  $v$  is the channel memory. A Viterbi decoder uses the Viterbi algorithm for decoding a bit stream that has been encoded using forward error correction based on a convolutional code. The Hamming distance is used as a metric for hard decision Viterbi decoders. The squared Euclidean distance is used as a metric for soft decision decoders. Many methods have been proposed to reduce the

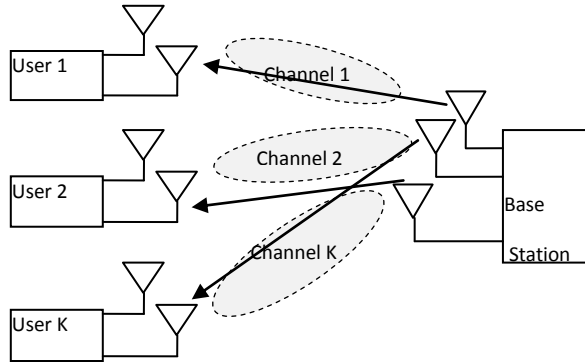
complexity of the Viterbi detector. One common method is to use a linear equalizer to shape the channel to one having a shorter length [5]–[7]. In some of these approaches, the target response to which Viterbi detector is matched is chosen as a short version of the infinite length response. However this choice is not good for the finite length case.

In this paper, the decoding techniques are discussed, as in MIMO system, there exist several receivers, such as maximum likelihood (ML) receiver, zero-forcing (ZF) receiver, minimum mean square error (MMSE) receiver and sphere decoding (SD) receiver, according to tradeoff between system performance and complexity.

## 2. SYSTEM MODEL

MU-MIMO is a MIMO system in which multiple users can take participation in data transmission simultaneously. In the uplink of cellular network, users transmit signals to the base station over the same channel but it is difficult for the base station to separate these signals. If transmitter provides channel feedback information back to the users then coordination among users may be possible. For this coordination each user must know channels experienced by other users as well as its own channel. In uplink, base station receives the data from multiple users. It is also known as uplink-MAC (multiple access channel). It is a multipoint to point communication.

In the downlink, base station transmits information simultaneously to a group of users. But there is some inter-user interference because signal received by one user will act as interference signal for other remaining users. It is also known as downlink-BC (broadcast). It is a point to multipoint communication.



**FIGURE 1:** Multi-User MIMO system model

The MU-MIMO system model uses  $M_{Tx}$ ,  $M_{Rx}$  antennas at transmitter and receiver side as shown in Figure 1. We considered MU-MIMO downlink system model in which base station transmits information simultaneously to a group of users. This system employs single base station equipped with  $M_{Tx}$  transmit antennas and  $K$  users where each user has  $M_{Rx}$  receive antennas. Let  $s_k$  denotes transmitted data intended for user  $k$ . For each user, symbol  $s_k$  is multiplied by  $c_k$ , thus the signal vector  $\mathbf{x} \in \hat{\mathbb{C}}^{M_{Rx} \times 1}$  can be written as

$$\mathbf{x} = \sum_{k=1}^K c_k s_k = \mathbf{C} \mathbf{s} \quad (1)$$

where  $\mathbf{C} = [c_1, c_2, \dots, c_K] \in \hat{\mathbb{C}}^{M_{Rx} \times K}$  is a beam forming matrix [1], and  $\mathbf{s} = [s_1, s_2, \dots, s_K]^{M_{Tx}} \in \hat{\mathbb{C}}^{K \times 1}$  is the signal vector. The received signal vector  $\mathbf{Y}_i$  of the  $i^{\text{th}}$  user is given as

$$\mathbf{Y}_i = \mathbf{H}_i \mathbf{x} + \mathbf{w}_i \quad (2)$$

$$\mathbf{Y}_i = \mathbf{H}_i \mathbf{C} \mathbf{s} + \mathbf{w}_i$$

$$\mathbf{Y}_i = \mathbf{H}_i \mathbf{C}_i \mathbf{s}_i + \sum_{k=1, k \neq i}^K \mathbf{H}_i c_k s_k + \mathbf{w}_i \quad (3)$$

where  $\mathbf{Y} = [y_1, y_2, \dots, y_K]^{M_{Tx}}$ ;  $\mathbf{s} = [s_1, s_2, \dots, s_K]^{M_{Tx}}$

$$\mathbf{H} = [(\mathbf{h}_1)^{M_{Tx}}, (\mathbf{h}_2)^{M_{Tx}}, \dots, (\mathbf{h}_K)^{M_{Tx}}];$$

$$\mathbf{C} = [c_1, c_2, \dots, c_K]; \mathbf{w} = [w_1, w_2, \dots, w_K]^{M_{Tx}}$$

The received signal vector  $\mathbf{Y}_i$  of the  $i^{\text{th}}$  user at the  $t^{\text{th}}$  symbol interval can be written as

$$\mathbf{Y}_i[t] = \mathbf{H}_i[t] \mathbf{C}[t] \mathbf{s}[t] + \mathbf{w}_i[t] \quad (4)$$

$$\mathbf{Y}_i[t] = \mathbf{H}_i[t] c_i[t] s_i[t] + \sum_{k=1, k \neq i}^K \mathbf{H}_i[t] c_k[t] s_k[t] +$$

$$\mathbf{w}_i[t] \quad (5)$$

The noise  $\mathbf{w}_i \in \hat{\mathbb{C}}^{(M_{Rx})_i \times 1}$  is independent complex Gaussian distributed with zero mean and unit variance. The MIMO channel  $\mathbf{H}$  for the  $i^{\text{th}}$  user is  $\mathbf{H}_i \in \hat{\mathbb{C}}^{(M_{Rx})_i \times (M_{Tx})_i}$ . MIMO channel is basically a realization of standard i.i.d Rayleigh fading channel [2].

### 3. DECODING TECHNIQUES IN MU-MIMO SYSTEM

There are two types of decoding techniques such as Linear and Non-Linear techniques.

#### 3.1 Linear Decoding Techniques-

Linear signal detection method treats all transmitted signals as interferences except for the desired stream from the target transmit antenna. Therefore, interference signals from other transmit antennas are minimized or nullified in the course of detecting the desired signal from the target transmit antenna.

##### 3.1.1 ZF (ZERO FORCING) Decoding Technique-

ZF receiver is one of the linear detectors. It nullifies the interference by the weight matrix:

$$\mathbf{W}_{ZF} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H$$

The other linear detector is MMSE receiver which in order to maximize the post-detection signal to interference plus noise ratio (SINR), the MMSE weight matrix is given as:

$$\mathbf{W}_{MMSE} = (\mathbf{H}^H \mathbf{H} + \sigma_z^2 \mathbf{I})^{-1} \mathbf{H}^H$$

##### 3.1.2 MMSE (MINIMUM MEAN SQUARE ERROR) Decoding Technique-

MMSE receiver requires the statistical information of noise  $\sigma_z^2$ . Noise enhancement effect in the course of linear filtering is significant when the condition number of the channel matrix is large, that is, the minimum singular value is very small.

#### 3.2 Non-Linear Decoding Techniques-

Non-linear detectors provide good results as compared to that of linear but with a little bit more complexity. Non-linear decoders are complex but have good BER performance.

##### 3.2.1 ML (MAXIMUM LIKELIHOOD) Decoding Technique-

ML receiver, which is known as an optimal receiver, detects the transmit symbol vector and is a set of all possible transmit symbol vectors [8], [9]. Since ML receiver detects proper transmit symbols by exhaustive search, it is difficult to evaluate an exact average error probability of ML receiver as a closed form [9], [10]. Therefore, the performance of ML receiver is analyzed by using average pairwise error probability (PEP) of two particular symbol vectors. Through this average pairwise error probability, we can know that ML receiver has the diversity order of and its performance is dominantly affected by the received minimum distance [8]. That is, as the received minimum distance is larger, the performance is better.

The ML detection calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel H, and finds the one with the minimum distance. If C and N<sub>T</sub> denote a set of signal constellation symbol points and a number of transmit antennas, respectively. Then, ML detection determines the estimate of the transmitted signal vector x as

$$\hat{x}_{ML} = \arg \min_{x \in C^N_T} \|y - Hx\|^2$$

The ML method achieves the optimal performance as the maximum a posteriori (MAP) detection when all the transmitted vectors are equally likely. However, its complexity increases exponentially as modulation order or the number of transmit antennas increases. The ML decoding technique is used previously as it provides good results but later on the SD is being used.

### 3.2.2 SD (SPHERE DECODING) Decoding Technique

SD is introduced originally by Finke and Pohst in [11] in 1985, this method intends to find the transmitted signal vector with minimum ML metric, that is, to find the ML solution vector. However, it considers only a small set of vectors within a given sphere rather than all possible transmitted signal vectors SD adjusts the sphere radius until there exists a single vector (ML solution vector) within a sphere. It increases the radius when there exists no vector within a sphere, and decreases the radius when there exist multiple vectors within the sphere. It is way better than all other techniques as it helps in improving the bit error rate and spatial complexity. Figure1 shows the idea behind the sphere decoding. Further in SD here are different algorithms used to achieve optimal results.

The calculation of the radius in the SD is the most difficult task, and it is to be done at the pre-processing level. If the radius is too large, average processing cycle becomes extremely high, making real time operation impossible. On the other hand, if the radius is too small, even the ML solution cannot satisfy the sphere constraint shown in figure 2. Thus setting the appropriate radius is very critical to successful implement the SD.

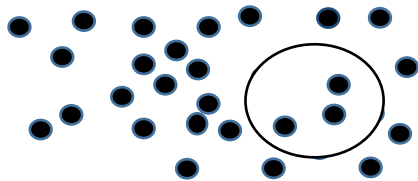


Fig2. Diagram of Sphere Decoding

SD algorithms for Spatial Modulation (SM) are developed to reduce the computational complexity of ML detectors. SD algorithms are a subset of decision feedback tree-search-decoders as shown in figure 3. They

perform the detection of MIMO data symbols by iterating through a detection tree, in which the tree levels, also referred to as dimensions, correspond to the elements of the received symbol. Those detectors differ basically in the way how they search along the tree. At this point, the difference between visited nodes and explored nodes should be pointed out. Any node that is not discarded is considered a visited node (VN). A subset of the VN are the explored nodes (EN). These are all VN with branching child nodes. The goal always is to visit and explore as little nodes as possible to keep the computational cost low. Various strategies exist to achieve this goal, all of which can or even have to be combined in order to work properly.

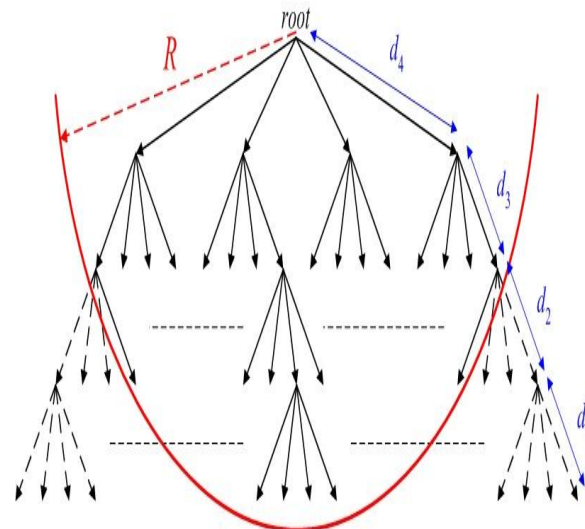


Fig.3 Tree structure for SD

The complexity of tree search algorithms is determined by two criteria

- The number of nodes that have to be examined and the operational cost per node. In SD, the number of visited nodes depends on the initial sphere radius and on the reduction of the radius constraints due to a radius update.
- The operational costs per node depends on the algorithm used.

SD is expressed as:

$$\arg \min_x \|y - \bar{H}x\|^2 = \arg \min_x (x - \hat{x})^T H^T H (x - \hat{x}) \quad \text{eqn1.}$$

The equation of SD can be derived by using the value of  $\hat{x}$  where

$\hat{x}$  is unconstrained least squared solutions i.e.  
 $\hat{x} = (H^H H)^{-1} H^H y$ , this relation exists for both real as well as complex systems.

H, y, x are used instead of  $\bar{H}$ ,  $\bar{y}$  &  $\bar{x}$  respectively.

Considering:

$$\begin{aligned} \|y - Hx\|^2 &= \|y - Hx - H\hat{x} + H\hat{x}\|^2 \\ &= (y - Hx - H\hat{x} + H\hat{x})^T (y - Hx - H\hat{x} + H\hat{x}) \end{aligned}$$

$$= \{(y - H\hat{x})^T + (H\hat{x} - Hx)^T\} \{(y - H\hat{x}) + (H\hat{x} - Hx)\} \quad \text{eqn.2}$$

$$= (y - H\hat{x})^T (y - H\hat{x}) + (H\hat{x} - Hx)^T (H\hat{x} - Hx) + (H\hat{x} - Hx)^T (y - H\hat{x}) + (y - H\hat{x})^T (H\hat{x} - Hx)$$

As,  $\hat{x}$  is the Least Squared Solution,

$$(H\hat{x} - Hx)^T (y - H\hat{x}) = (y - H\hat{x})^T (H\hat{x} - Hx) = 0$$

And thus the equation reduces to

$$\|y - Hx\|^2 = (y - H\hat{x})^T (y - H\hat{x}) + (H\hat{x} - Hx)^T (H\hat{x} - Hx) \quad \text{eqn.3}$$

Now substituting the value of  $\hat{x}$  with

$$(H^T H)^{-1} H^T y$$

eqn.3 becomes

$$= \{y - H (H^T H)^{-1} H^T y\}^T \{y - H (H^T H)^{-1} H^T y\} + (\hat{x} - x)^T H^T H (\hat{x} - x) \quad \text{eqn.4}$$

Since,

$$y - H (H^T H)^{-1} H^T y = \{I - H (H^T H)^{-1} H^T\} y,$$

the first term in the eqn.4 becomes

$$\begin{aligned} &= y^T \{I - H (H^T H)^{-1} H^T\}^T \{I - H (H^T H)^{-1} H^T\} y \\ &= y^T \{I - H (H^T H)^{-1} H^T\} \{I - H (H^T H)^{-1} H^T\} y \\ &= y^T \{I - H (H^T H)^{-1} H^T - H (H^T H)^{-1} H^T + H (H^T H)^{-1} H^T H (H^T H)^{-1} H^T\} y \\ &= y^T \{I - H (H^T H)^{-1} H^T\} y \quad \text{eqn.5} \end{aligned}$$

Which turns out to be constant with respect to  $x$ . From eqn.4 & 5, our relationship in eqn.1 immediately follows  $\arg \min_x \|y - Hx\|^2 = \arg \min_x (x - \hat{x})^T H^T H (x - \hat{x})$

This is the equation used for the calculation of the SD.

The Fincke-Phost (F-P), Schnorr-Euchner (S-E) and K-Best (KB) strategies are the computationally efficient means of realizing this enumeration [12], and so they have come to form the foundation of many existing SD [13], [14]. The fixed complexity sphere decoder (FSD) has been proposed to attain the near-optimal performance achieving the same diversity as the (ML).

Among many variations of the SD algorithm, SE-SD and KB-SD algorithms are the widely preferred implementation choice. KB-SD guarantees fixed throughput since it restricts the number of visited nodes per tree depth. However such restrictions cause severe performance degradation. Whereas, SE-SD is more attractive than KB-SD due to the reason that SE-SD can guarantee better error performance than KB-SD and it can be used together with a per block run time constraint to provide the fixed throughput without severe performance degradation.

Since, SD algorithms achieve the optimal bit error rate, it is widely adopted in wireless communication systems. However, complexity of SD and non-deterministic throughput makes implementation of SD hard. But still the research is being carried by the researchers on the SD, so that the problem of the complexity can be resolved.

## CONCLUSION

Till now SD is the best technique among all the decoding techniques. ZF and MMSE also provide the optimal results but the problem of the bit error rate remains as it is. By using some of the algorithms of these techniques we can achieve a little bit better results in terms of BER. In ML the BER and SNR both are improved not as much as it is required. In SD complexity is increased a bit but in the same technique the better and optimal results, which are required in a wireless communication system is also achieved.

## REFERENCES

- [1] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311-335, Mar. 1998.
- [2] I. Emre Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecomm.*, vol. 10, no. 6, pp. 585-595, Nov.-Dec. 1999.
- [3] R. L. Lupas and S. Verdú, "Near-far resistance of multiuser detection in asynchronous systems," *IEEE Trans. Commun.*, vol. 38, pp. 496-508, Apr. 1990.
- [4] Z. Xie, R. T. Short, and C. K. Rushforth, "A family of suboptimal detectors for coherent multiuser communications," *IEEE J. Select. Areas Commun.*, vol. 8, pp. 683-690, May 1990.
- [5] S. U. Qureshi and E. E. Newhall, "An adaptive receiver for data transmission over time-dispersive channels," *IEEE Trans. Inform. Theory*, vol. IT-19, pp. 448-457, July 1973.
- [6] J. Fitzpatrick, "Analysis of and coding for partial response magnetic recording systems," Ph.D. dissertation, Univ. California at San Diego, La Jolla, 1993.
- [7] D. D. Falconer and J. F. R. Magee, "Adaptive channel memory truncation for maximum likelihood sequence estimation," *Bell Syst. Tech. J.*, vol. 52, pp. 1541-1562, Nov. 1973.
- [8] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2003.
- [9] Z. Zou and R. D. Murch, "Performance analysis of maximum likelihood detection in a MIMO antenna system," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 187-191, Feb. 2002.
- [10] S. Talwar and A. Paulraj, "Blind separation of synchronous co-channel digital signals using an antenna array—Part II: Performance analysis," *IEEE Trans. Signal Process.*, vol. 45, no. 3, pp. 706-718, Mar. 1997.
- [11] U. Fincke and M. Pohst, "Improved methods for calculating vectors of short length in a lattice, including a complexity analysis," *Mathematics of Computation*, vol. 44, no. 170, pp. 463-471, Apr. 1985.
- [12] M. O. Damen, H. E. Gamal, and G. Caire, "On maximum-likelihood detection and the search for the closest lattice point," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2389-2402, October 2003.
- [13] A. M. Chan and I. Lee, "A new reduced-complexity sphere decoder for multiple antenna systems," in *IEEE International Conference on Communications*, vol. 1, April 2002, pp. 460-464.
- [14] B. Hassibi and H. Vikalo, "On the sphere decoding algorithm: Part I, The expected complexity," To appear in *IEEE Transactions on Signal Processing*, 2004